Admissible Rules of Many-Valued Logics

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Investigations of logical systems – in particular, many-valued logics – typically tend to focus on derivability. However, it can also be interesting and useful to "move up a level" and consider admissible rules of the system: the rules under which the set of theorems is closed. Whereas derivable rules (members of the consequence relation) may be thought of as providing an internal description of a logic, admissible rules provide an external perspective, describing properties. In algebra, such rules correspond to quasi-equations holding in free algebras, while from a computer science perspective, admissibility is intimately related to, and in certain cases may be reduced to, equational unification. For classical logic, derivability and admissibility coincide: the logic is structurally complete. However, for many non-classical logics – in particular, important intermediate, modal, many-valued, and substructural logics – this is no longer the case, and interesting questions arise as to the decidability, complexity, (finite) axiomatizability, and characterizations of their admissible rules (see, e.g., [3, 2, 1]).

The aim of this talk will be to explore the landscape of admissible rules and structural completeness for (fragments of) many-valued logics. In particular, various methods will be described for establishing structural completeness and its failure, and used to show for example that Gödel logic and product logic are structurally complete, while the implicational fragments of Łukasiewicz logic and basic logic are structurally complete, but not the full logics or indeed their implication-strong conjunction fragments. The historically pertinent case of the relevant logic RM (for which the disjunctive syllogism is admissible but not derivable) will be explored in detail and axiomatizations (bases) described for the admissible rules of various fragments.

References

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