Perfect GMV-Algebras and its Variety

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Every GMV-algebras appears as $\Gamma(G, u)$, [2], where (G, u) is a unital ℓ-group (not necessarily Abelian) with strong unit u and Γ is a generalized Mundici's functor [3] between the variety of GMV-algebras and the category of unital ℓ-groups

We define a category of perfect GMV-algebras, as GMV-algebras M having only one negation such that every maximal ideal of M is normal. We show that, for every perfect GMV-algebra M, there is a unique (up to isomorphism) ℓ-group G such that $M \cong \Gamma(\mathbb{Z} \times_{lex} G, (1, 0))$. Using the Holland representation of ℓ-groups and McCleary's Trichotomy Classification Theorem of primitive ℓ-groups we characterize the variety generated by perfect GMV-algebras. We show that this variety is generated by a unique perfect GMV-algebra $\Gamma(\mathbb{Z} \times_{lex} G, (1, 0))$, where G is a doubly transitive ℓ-group, and is characterized by a unique identity. In addition, we show that this variety coincides with the variety of GMV-algebras M such that every maximal ideal I is normal, and $M/I \cong \Gamma(\mathbb{Z}, 1)$. The results generalized those for MV-algebras [1].

[1] A. Di Nola, A. Lettieri, *Perfect MV-algebras are categorical equivalent to abelian ℓ-groups*, Studia Logica **53** (1994), 417–432.

[2] A. Dvurečenskij, *Pseudo-MV algebras are intervals of łgroups*, J. Australian Math. Soc. **72** (2002), 427–445.

[3] D. Mundici, *Averaging the truth-value in Łukasiewicz logic,* Studia Logica **55** (1995), 113–127.