

Weak MV-algebras

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MV-algebras were introduced in 50'ties by Chang as an algebraic semantics of the Łukasiewicz many valued sentential logic. A celebrated result of D.Mundici states that all MV-algebras can be derived from abelian l-groups.

This result became a starting point for a non-commutative generalization of MV-algebras introduced and studied by G.Georgescu and A.Iorgulescu as pseudo MV-algebras and independently by J.Rachunek as GMV-algebras. In principle, these are algebras with a binary operation and two unary operations (negations) coinciding whenever the binary operation is commutative. Similarly as for MV-algebras, A.Dvurecenskij proved that pseudo MV-algebras are intervals in l-groups.

Another important generalization of MV-algebras by omitting associativity but keeping commutativity was done by I.Chajda and J.Kuhr as NMV-algebras (non-associative MV-algebras). Here associativity was substituted by two weaker axioms allowing to prove that the induced relation remains transitive (and hence being an order relation). From a logical point of view, such a property is quite natural since in reasonable logics the set of truth values should be partially ordered.

For MV-algebras it is well known that their sections form MV-algebras again. However, this property is not true for NMV-algebras. It turned out that the sections of an NMV-algebra have a structure of an NMV-algebra iff it is an MV-algebra. The aim of our talk is to present another class of generalized MV-algebras, called WMV-algebras, admitting the same structure on sections.

These are shown to be equivalent to bounded lambda-lattices with section antitone involutions, form a variety which is regular and arithmetical. Implication reducts of WMV-algebras are recognized as section WMV-algebras with compatibility condition. Description of deductive systems and ideals of implication reducts will be also presented.