Generating Cut-free Ordinary Sequent Calculi for Logics Having Finite-Valued Semantics

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Logics with finite-valued semantics have found particularly many applications in a wide range of practical areas. The applicability of finite-valued semantics has recently been greatly increased by the use of non-deterministic matrices - shortly, Nmatrices - which interpret connectives of a logical language via multiple-valued functions on logical values. Nmatrices constitute a major generalization of the ordinary many-valued semantics for logical systems: while preserving all the fundamental advantageous properties of the ordinary finite-valued matrices, Nmatrices can be used to provide finite-valued semantics for many logics which do not possess semantics based on ordinary finite-valued matrices.

With the growing importance of finite-valued logics, a lot of efforts have been devoted to developing proof systems for such logics. For an ordinary n-valued semantics, the usual way of doing so in a uniform way is to employ a calculus of n-sequents, i.e., sequents with n components, or "sides" (or an equivalent deduction mechanism based on sets of n-signed formulas). Unfortunately, such mechanisms, though useful, are much less known and also more difficult to handle, or even understand, than ordinary, two sided sequents.

In this paper we show that for a large, central class of finite-valued logics one can transform a given n-sequent proof system into an equivalent calculus of ordinary sequents. The transformation is based on distinguishing between the individual logical values by checking whether a certain set of "unary" formulae specific for a given logical value have a particular arrangement of designated / non-designated values (whereby we do not distinguish between the individual designated or non-designated values) . Such designated/non-designated separation is then used to choose either the right-hand side or the left-hand side for the appropriate formula during translation to ordinary systems.

The simplest example of such a "distinguishing set of formulae can be found in the well-known Rosser-Turquette n-valued logics, where the logical value "i" is distinguished by the "J_i" operator, which has a designated value for "i" only. However, the use of other, usually richer "distinguishing sets

allows application of the method to a wide class of logics having sufficiently expressive languages. Moreover: the resulting transformation preserves the structures of proofs in the original calculus. In particular: it preserves admissibility of the cut rule, thus allowing us to get cut elimination for the resulting, two-sided calculus for free.

The present work is a sequel and companion to [1], where we showed a general method of developing proof systems for propositional logics based on Nmatrices. Accordingly, the two papers combined provide a method for developing ordinary sequent calculi for a large class of finite-valued logics based on Nmatrices.

In [1] we illustrated the transformation of n-sequent calculi into ordinary calculi discussed here on the examples of certain important logics, and announced a forthcoming general result and method - which we give here, together with the relevant proofs and further examples of its application to many-valued logics. A particularly interesting one is a new application to information sources logics, where we consider a number of sources providing information (which might be incomplete or inconsistent) about truth or falsity of formulas, and a processor which combines this information according to certain rules.

1. A. Avron and B. Konikowska, Multi-valued Calculi for Logics Based on Non-determinism, Proceedings COS'04 , Vienna 2004, Journal of the Interest Group in Pure and Applied Logic, 2005 (10), pp. 365-387.