EQ-algebra for fuzzy type theory

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The fuzzy type theory (FTT) uses one of the common algebras considered as algebras for truth values for various kinds of fuzzy logic, namely IMTL_Δ, Łukasiewicz$_\Delta$, BL_Δ or ŁΠ-algebra. However, the basic connective in FTT is fuzzy equality. Therefore, we try in this paper to introduce a special algebra that we will call EQ-algebra and that reflects directly the syntax of FTT.

The following is a preliminary definition of EQ-algebra. It is an algebra

$$< L, \wedge, \otimes, \sim>$$

of type (2, 2, 2) where:

(i) $< L, \wedge >$ is a commutative idempotent semigroup.

(ii) $< L, \otimes >$ is a commutative semigroup.

(iii) There is $\mathbf{1} \in L$ such that for all $a \in L$

$$(a \wedge \mathbf{1}) \sim a = \mathbf{1}.$$

(iv) $a \sim b = b \sim a$ for all $a, b \in L$.

(v) $a \wedge \mathbf{1} = a \otimes \mathbf{1} = a$ for all $a \in L$.

(vi) For all $a, b \in L$

$$(((a \sim b) \otimes (b \sim c)) \wedge (a \sim c)) \sim ((a \sim b) \otimes (b \sim c))).$$

(vii) $(a \wedge \mathbf{1}) \sim \mathbf{1} = a$ for all $a \in L$.

Some consequences of these axioms and further extensions will be discussed and the relation of EQ-algebra to the other mentioned algebras will be outlined.