Special Fuzzy Predicate Logic Theory for Rule-Based Systems

Irina Perfilieva
University of Ostrava, Czech Republic
Irina.Perfilieva@osu.cz

The evolution of a theory of fuzzy IF-THEN rules and its contribution to the establishment of fuzzy logic is a focus of this paper. I am advocating in favour of Hájek's fuzzy logic regarding it as a right methodology for special logical theories. A theory of fuzzy IF-THEN rules (as a special theory in the above considered sense) is proposed. This theory aims to create a system of fuzzy IF-THEN rules free of conflicts (logically consistent) and rich enough to be able to make non-trivial conclusions or answer inquires.

We claim that the system of IF–THEN rules represents a certain dependence between parameters x and y, similar to tabular definition of a function. The formally expressed dependence becomes meaningful after appropriate interpretation. Let us propose to interpret the system by a fuzzy relation R which may not be constructed directly from the rules on the basis of their logical structure and on the principle of truth functionality.

Let a BL-algebra be chosen to express the relationship between a system of fuzzy IF-THEN rules and its interpretation.

We are going to construct a special predicate theory of fuzzy IF-THEN rules. For the basic predicate calculus we chose the Hájek's BL$\forall$.

Suppose that our language $\mathcal{J}_n$, $n \geq 1$, is extended by special unary predicate symbols $A_1, \ldots, A_n \in \mathcal{P}$ and $B_1, \ldots, B_n \in \mathcal{P}$, and a special binary predicate symbol $R \in \mathcal{P}$.

The notions of *term* and *formula* are defined as in the classical predicate logic with the following additional abbreviation: if $\varphi(x), \psi(x, y)$ are formulas where x is a free variable then the following construction is a formula too:

$$(\varphi \circ \psi)(y) = (\exists x)(\varphi(x) \& \psi(x, y)).$$

The *special theory* $\mathcal{R}_n$ of n IF-THEN rules consists of:

- all axioms of the Hájek's BL$\forall$,
- special axioms:
  SA1 $R(x, y) \to \bigwedge_{i=1}^{n} (A_i(x) \to B_i(y))$
  SA2$_i$ $B_i(y) \to (A_i \circ R)(y)$
  where $i = 1, \ldots, n$ (so that we have n axioms of type **SA2**),
- deduction rules:
  MP (modus ponens): from φ, and $\varphi \to \psi$ infer ψ,

Gen (generalization): from φ infer $(\forall x)\varphi$,
CRI: from $\varphi(x)$, and $\psi(x, y)$ infer $(\varphi \circ \psi)(y)$.

We present some examples of provable formulas. Let A be an arbitrary unary predicate, then for each $i = 1, \dots, n$, $\mathcal{R}_n$ proves the following:

$$(\forall y)(B_i(y) \equiv (A_i \circ \bigwedge_{i=1}^{n}(A_i \to B_i))(y)), \tag{1}$$

$$R(x, y) \to (A(x) \to (A \circ R)(y)), \tag{2}$$

$$(\forall x)(A(x) \equiv A_i(x)) \to (\forall y)((A \circ R)(y) \equiv B_i). \tag{3}$$

Logical theory of fuzzy IF-THEN rules aims to make inferences with antecedents different from those formalized by A_i (cf. (2)). For this purpose, the deduction rule **CRI** has been proposed. However, if we cannot relate a given antecedent, say A, to any of A_i, the conclusion, obtained by the deduction, is so general that it does not express any specific property or constraint. In this case, some additional information (in the form of a new fuzzy IF-THEN rule) is required. Logically, this means that we would like to extend our special theory by adding new special axioms. This must be done carefully, keeping the consistency of the original theory. In this contribution, different definitions of consistency are considered and compared.