Truth values on generalizations of fuzzy structures

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Inspired by the considerations of L. Zadeh, P. Hájek formalized the fuzzy truth value "very true". He enriched the language of the basic fuzzy logic BL by adding a new unary connective vt and introduced the (propositional) logic BL_{vt}. He proved the completeness of BL_{vt} by using the so-called BL_{vt}-algebras, an algebraic counterpart of BL_{vt}. Recall that a BL_{vt}-algebra is a BL-algebra with a unary operation (an operator) which is, among others, subdiagonal.

Very recently, V. Vychodil introduced a formalization of the fuzzy truth value "slightly true" related to "very true" by enriching the language of BL_{vt} by another unary connective st and defined the (propositional) logic $BL_{vt,st}$. He introduced the so-called $BL_{vt,st}$-algebras and, analogously as Hájek for BL_{vt}, he used them to prove the completeness of $BL_{vt,st}$. A $BL_{vt,st}$-algebra is a BL_{vt}-algebra with a unary operation st which is, among others, superdiagonal.

Bounded commutative residuated lattice ordered monoids ($R\ell$ monoids) are algebraic structures which generalize, e.g., both BL-algebras and Heyting algebras (an algebraic counterpart of the intuitionistic propositional logic). Nevertheless, many of properties of BL-algebras are also satisfied in any bounded commutative $R\ell$ monoids. Therefore bounded commutative $R\ell$ monoids could be taken as an algebraic semantics of a more general logic than Hájek's fuzzy logic.

We introduce $R\ell_{vt}$-monoids as bounded commutative $R\ell$ monoids with a unary subdiagonal and monotone self-mapping (called a vt-operator) which generalize BL_{vt}-algebras. Further, we derive from the vt-operators, in a purely algebraic and uniform way, subdiagonal and monotone self-mappings, called at-operators (at = almost true) which are dual to vt-operators and define $R\ell_{at}$-monoids. We study properties of $R\ell_{vt}$- and $R\ell_{at}$-monoids and give some connections between them.