SUBJECTIVE PROBABILITY BETWEEN CLASSICAL AND NON-CLASSICAL LOGICS

HYKEL HOSNI

Under normal circumstances, agents in the "real world" reason and act on the basis of information which falls short of being perfect. There are many ways in which an agent's information can be said to be imperfect: it can be incomplete (or partially specified), vague, contradictory, unreliable and so on. Among those phenomena, however, particular importance has been attached to the problem of characterising reasoning under incomplete information, or as it is customary to say, *uncertain reasoning*, which can be described in very general terms as the problem of defining which properties an agent's *belief function* should satisfy in order for such an agent to be capable of reasoning "rationally".

During the 1930's, de Finetti, and independently Ramsey gave fundamental suggestions (which have been subsequently refined in the 1950's by Kemeny and Shimony) concerning the mathematical constraints that such belief functions should satisfy by introducing the so-called Dutch Book Argument. In a nutshell, the argument establishes that if an agent is to avoid obviously irrational patterns of behaviour, its belief function should satisfy the constraints which define a probability function and, conversely, if its belief function is a probability function, then no obviously irrational belief assessment can ever arise. For this argument, de Finetti developed his operational definition of (degree) of probability in terms of an agent's willingness to bet on a certain betting scheme. Key to the argument is the definition of an "obviously irrational belief assessment" as one which could lead an agent to loose no matter what the outcome of its bets would turn out to be – in other words an irrational agent is one who is open to be "dutch-booked". Due to its intuitiveness, the Dutch Book Argument provides a very strong reason for interpreting an agent's belief function (its degrees of belief) in terms of (subjective) probability.

De Finetti referred to the "no-Dutch Book" condition as the property of *coherence* for belief functions, thus introducing in his characterisation of uncertain reasoning a logical criterion. Despite this fact, and despite his referring to probability as the "logic of the uncertain", de Finetti did not conceive of his work as belonging to mathematical logic. In fact he never refrained from criticizing – often in his highly idiosyncratic manner – the excessive constraints imposed on probabilistic reasoning by purely logical considerations (take, for instance the case of σ additivity). In a sense it can be said that de Finetti aimed at a minimally-logical formulation of uncertain reasoning.

The advent of logic-based artificial intelligence in the early 1970's however, lead naturally to the attempt of extending de Finetti's characterisation of uncertain reasoning in the direction of a *probability logic* intended as an extension of classical logic capable of enriching this latter with the expressive power required to characterise rational inferences under uncertainty. Yet, de Finetti's subjectivistic

theory of probability, if interpreted logically, is essentially based on classical logic, so that the probability logic programme faces a number of non-trivial methodological problems. The aim of this paper is to formulate explicitly and then discuss some of those issues. Given the vastity of the field we shall restrict ourselves to a small number of particularly important points.

First of all we shall review de Finetti's notion of coherence arguing that in the context of classical logic it can easily be seen as being too conservative (i.e. restrictive). To this end we shall illustrate the framework of Maximum Entropy reasoning which provides a refinement of de Finetti's framework in the direction of rational inference under uncertainty. The basic idea here is that once coherence has been satisfied, further logical principles can be put to work in order to restrict the space of possible probability distributions over an agent's beliefs.

Secondly we shall discuss the methodological issues involved in extending the subjectivistic account of uncertain reasoning to more general semantics than the classical two-valued one. The key notion to be discussed here is the definettian concept of event on which his whole account of probability is based. In his original formulation of the betting scheme, de Finetti regarded an event – the object of an agent's probability assessment – as a three-valued object: true, false, undefined. In this latter case probability itself ceases to be defined. In the light of this we shall consider the problem of defining (in de Finetti's sense) probability on a class of *fuzzy events*.

If time permits, we shall finally speculate on some reasons why de Finetti would have not, probably, liked the whole programme of probability logic.

Scuola Normale Superiore, Pisa
E-mail address: `hykel.hosni@sns.it`