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ERGODIC THEOREM FOR ERGODIC MAPPING ON B-STRUCTURES

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In the contribution we will show extended ergodic theorem for B-structures with a state. Classic ergodic theorem is determined for ergodic mapping on Ω, where (Ω, S, P) is probability space and $\xi : \Omega \to R$ is integrable random variable. In our case is Ω replaced by B-structure B, which is defined as a system $(B, \oplus, \leq, 0_B, 1_B)$ such that:

(i) $\oplus$ is a partial binary operation on B;

(ii) $\leq$ is a partial ordering on B;

(iii) 0_B is the smallest, 1_B is the largest element in $(B, \leq)$.

Instead of integrable random variable $\xi : \Omega \to R$ we use integrable observable on B. For observable we will take a mapping $x : \mathcal{B}(R) \to B$, which satisfy the following statements:

(i) $x(R) = 1_B$, $x(\emptyset) = 0_B$; (ii) $A, B \in \mathcal{B}(R)$ and $A \cap B = \emptyset$ then $x(A \cup B) = x(A) \oplus x(B)$;

(iii) if $A_n \in \mathcal{B}(R)$:$A_n \nearrow A$ then $x(A_n) \nearrow x(A)$.

It is integrable if exists $\int\limits_R t dm_x(t)$, where $m_x = m \circ x$ with m as a state on B.

So the ergodic theorem is:

Let x be integrable observable on B-structure B with state m, for which the following holds: $\forall a \in B : m(\lambda(a)) = m(a)$ and $\lambda : B \to B$ is ergodic mapping. Then the sequence $(y_n)_{n=1}^{\infty}$ defined by a formula: $y_n = \frac{1}{n} \sum\limits_{i=0}^{n-1} \lambda^i \circ x - E(x)$ converges m-almost everywhere to 0.

References

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