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RECENT PROGRESS IN IF-PROBABILITY THEORY

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In the communication two new concepts are presented for the family $\mathcal{F}$ of all IF-events, i.e. pairs $A = (\mu_A, \nu_A)$ of measurable functions $\mu_A, \nu_A : \Omega \to [0,1]$ such that $\mu_A + \nu_A \leq 1$. If one uses the Lukasiewicz connectives

$$A \oplus B = ((\mu_A + \mu_B) \wedge 1, (\nu_A + \nu_B - 1) \vee 0)$$

then by a convenient embedding (see [5]) the problem of probablity on $\mathcal{F}$ can be reduced to the well developped probability theory on MV-algebras ([6]). Recall that a partial ordering is defined by the equivalence relation

$$A \leq B \Longleftrightarrow \mu_A \leq \mu_B, \nu_A \geq \nu_B$$

φ-probability

The first extension of the method is the study of the case, when instead of Lukasiewicz the φ-connectives are used

$$A \oplus_\varphi B = (\varphi^{-1}(\varphi(\mu_A) + \varphi(\mu_B)) \wedge 1, 1 - (\varphi^{-1}(\varphi(1-\nu_A) + \varphi(1-\nu_B)) \wedge 1))$$
$$A \odot_\varphi B = ((\mu_A + \mu_B - 1) \vee 0, (\nu_A + \nu_B) \wedge 1))$$

This approach preserves all good properties of the MV-algebra probability theory and contains a large class of special cases ([4]). Denote by$\mathcal{F}$ the family of all IF-events. Similarly as in the classical case and in the fuzzy case and in the quantum case, a probability (or a state) has been introduced as a mapping $m : \mathcal{F} \to [0,1]$ being continuous, additive and satisfying some boundary conditions. Here the main difference is in additivity:

$$m(A) + m(B) = m(A \oplus B) + m(A \odot B)$$

Generally there are infinitely many possibilities how to define additivity

$$m(A) + m(B) = m(S(A,B)) + m(T(A,B))$$

where

$$S(A,B) = (S(\mu_A, \mu_B), T(\nu_A, \nu_B))$$
$$S, T : [0,1]^2 \to [0,1]$$

being such binary operations that

$$S(u,v) + T(1-u, 1-v) \leq 1.$$

Of course, the IF-probability theory cannot be reduced to the fuzzy probability theory defined on a tribe. There are two reasons. First, probability on $\mathcal{F}$ cannot be reduced to a probability on the corresponding tribe $\mathcal{T}$ of fuzzy sets because of the representation theorems ([5]). Secondly, the Kolmogorov probability theory has 3 fundamental notions: probability, random variable, and expectation. In our fuzzy case an analogous situation occurs. And there exists no aparatus in fuzzy probability concept corresponding to random variables in the Kolmogorov theory or corresponding to observables in the quentum structure probability theory. Therefore also in our $\mathcal{F}$ case it is necessary to built the observables theory.

The main results: existence of joint φ-observable, limit theorems of probability theory for squences of independent φ-observables.

Probability on B-structures

The second approach, theory of of probability on B-structures is much more general ([3]), it is a quintuple $(B, \widehat{\oplus}, \leq, 1_B, 0_B)$, where $\widehat{\oplus}$ is a partial binary operation on B, $\leq$ is a partial ordering on B with the greatest element 1_B and the least element 0_B. Again a state is a mappping $m : B \to [0,1]$, continuous, satisfying the boundary conditions and additive:

$$a = b\widehat{\oplus}c \Longrightarrow m(a) = m(b) + m(c).$$

The structure seems to be not interesting from the algebraic point of view. On the other hand, if one consider an observable $x : \mathcal{B}(R) \to B$ than the composite mapping $m_x = m \circ x$ is a classical probability measure. Therefore it is possible to extend also for B-structures the method of local representation successfully used in the MV-algebra probability theory and some important assertions holds even in this case, e.g. Central limit theorem, Strong law of large numbers and Weak law of large numbers.

Of course, it is not possible to prove the existence of the joint observable as was possible in the φ-probability theory. The problem of the existence of the joint observable must be studied separately in some concrete situations.

Recall that also in th MV-algebra case the existence of the joint observable is assumed in the formulation of independency of a sequence (x_n) of observables and only in some special MV-algebras the existence of the joint observable can be proved.

References

[1] Atanassov, K.: Intuitionistic Fuzzy sets: Theory and Applications. Physica Verlag. New York 1999.

[2] Atanassov, K., Riečan, B.: On two new types of probability on IF-events. Submitted to Notes on IFS.

[3] Čunderlíková - Lendelová, K., Riečan, B.: Probability theory on B-structures. Submitted to Fuzzy Sets and Systems.

[4] Renčová, M.: On the φ-probability and φ-observables. Submitted to Fuzzy Sets and Systems.

[5] Riečan, B.: Probability theory on IF-events. In: Algebraic and Proof-Theoretic Aspects of Non-calssical Logics. Papers in Honor of Daniele Mundici on the Occasion of His 60th Birthday. Springer Lecture Notes in Artificial Intelligence, Springer, Berlin 2007, pp. 290 -308.

[6] Riečan, B., Mundici, D.: Probability on MV-algebras. In: Handbook on Measure Theory (E.Pap ed.), Elsevier, Amsterdam 2002, pp. 869 - 909.

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