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LYAPUNOV DECOMPOSITION OF MEASURES ON EFFECT ALGEBRAS

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Effect algebras have been introduced by D.J. Foulis and M.K. Bennett in 1994 (see [B-F]) for modelling unsharp measurement in a quantum mechanical system. They are a generalization of many structures which arise in quantum physics (see [B-C]) and in Mathematical Economics (see [B-K] and [E-Z]), in particular of orthomodular lattices in non-commutative measure theory and MV-algebras in fuzzy measure theory. After 1994, there have been a great number of papers concerning effect algebras. We refer to [D-P] for a bibliography.

In 1974 Kluvanek and Knowles (see [K-K]) proved a decomposition theorem of a closed vector measure on a σ-algebra into the sum of a Lyapunov measure (i.e. with convex range on every interval) and an "anti-Lyapunov" measure. This decomposition theorem was based on a characterization of Lyapunov vector measures on σ-algebras given by Knowles in [K] in 1974. Such a characterization involves the integral map, which is not defined in effect algebras.

In 1991 De Lucia and Wright in [D-W] proved a characterization of closed Lyapunov measures on σ-algebras by a different condition, namely non-injectivity (i.e. for every non-negligible a, there exist $b, c \leq a$ with $b \triangle c$ non-negligible and $\mu(b) = \mu(c)$). This result of De Lucia and Wright has been extended in 2003 by Avallone and Barbieri in [A-B] to modular measures on D-lattices, by replacing non-injectivity with another condition, called pseudo non-injectivity, which is equivalent to non-injectivity for measures on σ-algebras. Precisely μ is pseudo non-injective if for every non-negligible a, there exist orthogonal non-negligible elements b, c with $b \oplus c \leq a$ and $\mu(b) = \mu(c)$.

Starting from the result of Avallone and Barbieri, in this paper we prove the following Kluvanek-Knowles type decomposition theorem: Every closed modular measure on a σ-complete D-lattice (i.e. a lattice ordered effect algebra) can be decomposed into the sum of a Lyapunov modular measure and an "anti-Lyapunov" modular measure.

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