Degrees of Truth and Epistemic States

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In quite a few, sometimes influential, works dealing with knowledge representation, there is a temptation to extend the truth-set underlying a given logic with values expressing ignorance and contradiction. It then leads to some truth-functional many-valued logic different from the original one, but often using the same syntax. This is the case for instance with partial logic and Belnap bilattice logic with respect to classical logic. It is found again in interval-valued, and type two extensions of fuzzy sets.

This talk insists that neither ignorance nor contradiction cannot be viewed as additional truth-values nor processed in a truth-functional manner, and that doing it leads to weak or debatable uncertainty handling approaches. A similar difficulty is also found in three- valued logics of rough sets.

In fact, partial ignorance and contradiction are meta-level notions, like deduction and consistency and refer to epistemic states. When using the same language for modeling epistemic and objective notions, there is no way to tell the case of not believing a proposition from believing its contrary, not to tell the case of knowing that one among several propositions is true from the case of knowing that their disjunction is true. Modal logics have been used to handle epistemic notions in a more satisfactory way, but the Kripke semantics based on relations over possible worlds are perhaps unnecessarily complex to handle plain notions of epistemic states. A simpler semantics can be based on sets of epistemic states understood as non-empty subsets of interpretations.

We suggest that, in order to handle epistemic notions at the language level, we need two-tiered systems where one logic (typically propositional logic) describing events is encapsulated by another one (that can be many-valued) describing beliefs or testimonies. Such an approach paves the way to a general approach to logics of uncertainty, along the lines suggested by Esteva, Godo and Hájek casting probability, possibility or belief functions within a suitable multiple-valued setting, where beliefs are graded, but events remain Boolean.

References

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