Belief Functions, Möbius Transform and Integral Representations on MV-algebras

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Probability on MV-algebras was developed in the papers by Mundici and Riečan. Among many investigated facets of this theory count conditiong or bookmaking over events represented by formulas in infinite-valued Łukasiewicz propositional calculus. A characterization relating many-valued probability to classical probability on Boolean algebras is the following: probability (state) on an MV-algebra is just Lebesgue integral with respect to a uniquely determined Borel probability measure on the maximal ideal space.

The condition of additivity from the definition of state can be relaxed to study more general real functionals on MV-algebras. Non-additive set functions, such as belief functions, possibility measures and upper probabilities, appear already on Boolean algebras in connection with game theory or statistics. The whole class of such imprecise probabilities on MV-algebras is studied in the recent paper by Fedel, Keimel, Montagna and Roth.

In this contribution we will single out *belief functions* on MV-algebras. In particular, we will show that every belief function

- is *totally monotone* with respect to the lattice reduct of the MV-algebra,
- is characterized through its *Möbius transform* which is a state on a certain MV-algebra constructed on the ideal space,
- is represented by *Choquet integral* with respect to a unique belief function on the maximal ideal space of the MV-algebra.